Math Virtual Learning

Calculus AB

Derivatives of Special Functions

May 6, 2020

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Lesson: May 6, 2020

Objective/Learning Target:

Lesson 3 Derivatives Review

Students will calculate derivatives of special functions such as ln(x), e^x, inverse trig functions.

Warm-Up:

Note: This is a review. For more examples refer back to your notes.

Watch Videos: Derivative of ln(x)

Derivative of e^x

Derivative of inverse trig functions

Notes:

The function $f(x)$	The derivative $\frac{df}{dx}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
a^x	$a^x \ln a$
$\log_a x$	$\frac{1}{x \ln a}$

Derivative	Domain
$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$	$-1 < x < 1$
$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$	$-1 < x < 1$
$(\arctan x)' = \frac{1}{1+x^2}$	$-\infty < x < \infty$
$(\text{arccot}\, x)' = -\frac{1}{1+x^2}$	$-\infty < x < \infty$
$(\text{arcsec}\, x)' = \frac{1}{\lvert x \rvert \sqrt{x^2-1}}$	$x \in (-\infty,-1) \cup (1,\infty)$
$(\text{arccsc}\, x)' = -\frac{1}{\lvert x \rvert \sqrt{x^2-1}}$	$x \in (-\infty,-1) \cup (1,\infty)$

Examples:

Find the derivative of

$$f(x) = \ln(3x - 4)$$

Solution

We use the chain rule. We have

$$(3x - 4)' = 3$$

and

$$(\ln u)' = 1/u$$

Putting this together gives

$$f'(x) = (3)(1/u)$$

$$= \frac{3}{3x - 4}$$

Find the derivative of

$$f(x) = 2^x$$

Solution

We write

$$2^x = e^{x \ln 2}$$

Now use the chain rule

$$f'(x) = (e^{x \ln 2})(\ln 2) = 2^x \ln 2$$

Examples:

(c) $y = \dfrac{5\mathbf{e}^{x}}{3\mathbf{e}^{x}+1}$ *Hide Solution* ▾

We'll need to use the quotient rule on this one.

$$\begin{aligned} y' &= \frac{5\mathbf{e}^{x}\left(3\mathbf{e}^{x}+1\right)-\left(5\mathbf{e}^{x}\right)\left(3\mathbf{e}^{x}\right)}{\left(3\mathbf{e}^{x}+1\right)^{2}} \\ &= \frac{15\mathbf{e}^{2x}+5\mathbf{e}^{x}-15\mathbf{e}^{2x}}{\left(3\mathbf{e}^{x}+1\right)^{2}} \\ &= \frac{5\mathbf{e}^{x}}{\left(3\mathbf{e}^{x}+1\right)^{2}} \end{aligned}$$

Examples:

$$y = \arctan \frac{1}{x}$$

Solution.

By the chain rule,

$$y' = \left(\arctan \frac{1}{x} \right)' = \frac{1}{1 + \left(\frac{1}{x}\right)^2} \cdot \left(\frac{1}{x} \right)' = \frac{1}{1 + \frac{1}{x^2}} \cdot \left(-\frac{1}{x^2} \right) = -\frac{x^2}{(x^2+1)\, x^2}$$

$$= -\frac{1}{1 + x^2}.$$

Examples:

$$y = \operatorname{arccot} x^2$$

Solution.

Using the chain rule, we have

$$y' = (\operatorname{arccot} x^2)' = -\frac{1}{1 + (x^2)^2} \cdot (x^2)' = -\frac{2x}{1 + x^4}.$$

Practice:

1) Calculate the derivative of the following:

$$f(x) = 3e^x + 10x^3 \ln x$$

2) Calculate the derivative of the following:

$$y = \arcsin(x - 1)$$

Answer Key:

Once you have completed the problems, check your answers here.

1)

$$f'(x) = 3e^x + 30x^2 \ln x + 10x^3\left(\frac{1}{x}\right)$$

$$= 3e^x + 30x^2 \ln x + 10x^2$$

2) *Solution.*

$$y' = (\arcsin(x-1))' = \frac{1}{\sqrt{1-(x-1)^2}} = \frac{1}{\sqrt{1-(x^2-2x+1)}}$$

$$= \frac{1}{\sqrt{\cancel{1} - x^2 + 2x - \cancel{1}}} = \frac{1}{\sqrt{2x-x^2}}.$$

Additional Practice:

Interactive Practice

More Interactive Practice

Extra Practice with Answers